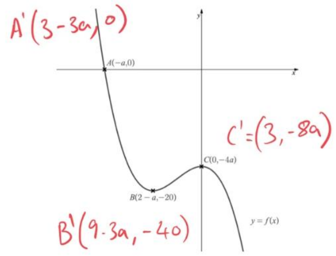
Q1

1

The diagram below shows the graph of $y = \mathrm{f}(x)$.
The stationary points and intercepts with the coordinate axes are marked on the diagram.



On separate diagrams, sketch the graphs with the following equations:

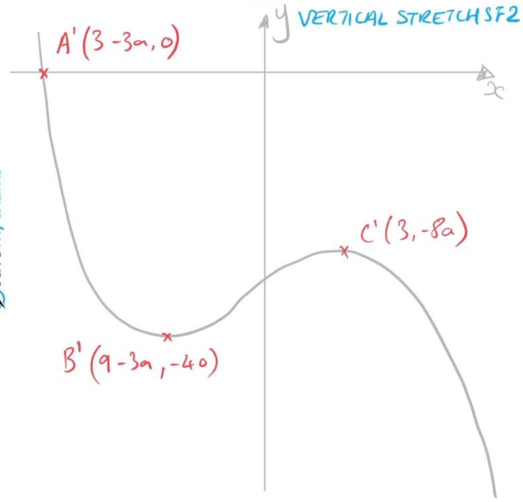
(i) $y = 2\mathrm{f}\left(\frac{1}{3}x - 1\right)$,
(ii) $y = -2\mathrm{f}(x + 1) + 1$.

On each diagram, mark the coordinates of the images of the points A, B and C under the given transformation, giving your coordinates in terms of a.

[6]

i)

TRANSLATION $\begin{pmatrix}1\\0\end{pmatrix}$
HORIZONTAL STRETCH SF3
VERTICAL STRETCH SF2



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ii) TRANSLATION $\begin{pmatrix}-1\\0\end{pmatrix}$

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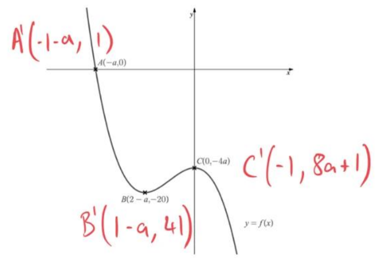

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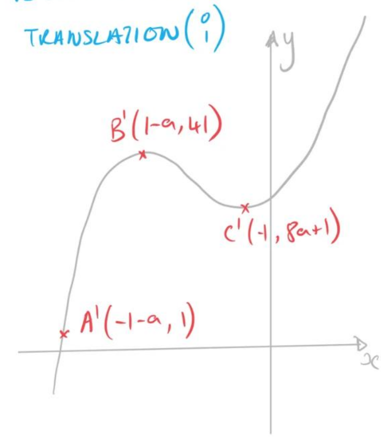
[6]

ii) TRANSLATION $\begin{pmatrix}-1\\0\end{pmatrix}$
VERTICAL STRETCH AND REFLECTION SF-2
TRANSLATION $\begin{pmatrix}0\\1\end{pmatrix}$



Q2a

2a

(a) Describe, in order, a sequence of transformations that would map the graph of $y = \mathrm{f}(x)$ onto each of the following graphs:

(i) $y = a\mathrm{f}(x + b) + c$ for the case when $a > 0$
(ii) $y = -\mathrm{f}(-x)$

[4]

(b) How, if at all, would your answer to part (a) (i) change if $a = 1$ or if $a < 0$?

[2]

a)

i) TRANSLATION $\begin{pmatrix} -b \\ 0 \end{pmatrix}$

VERTICAL STRETCH SF a

TRANSLATION $\begin{pmatrix} 0 \\ c \end{pmatrix}$

ii) REFLECTION IN y AXIS $(x = 0)$

REFLECTION IN x AXIS $(y = 0)$

Q2b

2b

(a) Describe, in order, a sequence of transformations that would map the graph of $y = \mathrm{f}(x)$ onto each of the following graphs:

(i) $y = a\mathrm{f}(x + b) + c$ for the case when $a > 0$
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[4]

(b) How, if at all, would your answer to part (a) (i) change if $a = 1$ or if $a < 0$?

[2]

b)

$a = 1$ VERTICAL STRETCH SF 1

WHICH WOULD HAVE NO EFFECT SO COULD BE LEFT OUT

$a < 0$ WOULD INTRODUCE A VERTICAL REFLECTION IN x AXIS

Q3

3

Show that the graph of $y = \mathrm{p}(x)$ where $\mathrm{p}(x) = 2x + 1$ maps onto the graph of its inverse under the transformations described by $\frac{1}{2}\mathrm{p}\left(\frac{1}{2}x\right) - 1$.

[4]

FIND INVERSE $p^{-1}(x)$

LET $y = 2x + 1$

$\frac{y-1}{2} = x$ $\quad p^{-1}(x) = \frac{x-1}{2}$

$= \frac{1}{2}(x-1)$

APPLY TRANSFORMATIONS TO $p(x)$

$\frac{1}{2}p\left(\frac{1}{2}x\right) - 1 \Rightarrow \frac{1}{2}\left[\cancel{2}\left(\frac{1}{\cancel{2}}x\right) + 1\right] - 1$

$\frac{1}{2}(x+1) - 1$

$\frac{1}{2}x + \frac{1}{2} - 1$

$\frac{1}{2}x - \frac{1}{2} = \frac{1}{2}(x-1)$

$\boxed{\frac{1}{2}p\left(\frac{1}{2}x\right) - 1 = p^{-1}(x)}$

Q4

4

Prove, that for a constant k, $k \neq 0$, if $\mathrm{f}(x) = kx$, then $\mathrm{f}^{-1}(x) = \frac{1}{k}\mathrm{f}\left(\frac{1}{k}x\right)$.

[4]

FIND INVERSE $f^{-1}(x)$

LET $y = kx$

$\frac{y}{k} = x$ $\quad f^{-1}(x) = \frac{x}{k}$

APPLY TRANSFORMATIONS TO $f(x)$

$\frac{1}{k}f\left(\frac{1}{k}x\right) \Rightarrow \frac{1}{k}\left[\cancel{k}\left(\frac{1}{\cancel{k}}x\right)\right]$

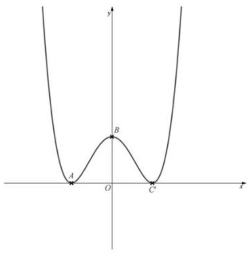
$\frac{1}{k}[x] = \frac{x}{k}$

$\boxed{f^{-1}(x) = \frac{1}{k}f\left(\frac{1}{k}x\right)}$

Q5a

5a

A sketch of the graph with equation $y = f(x)$, where $f(x) = (x^2 - a)^2$, with $a > 1$ is shown below.



The points A, B and C are points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A, B and C.

[2]

(b) Sketch the graph of $y = -\frac{1}{2}f(x-1)$, labelling the images of the three points A, B and C and stating their coordinates in terms of a.

[3]

(c) Suggest, in terms of $f(x)$, a combination of at least two transformations, such that the points A, B and C transform to new positions but remain lying on their respective axes.

[2]

a) $x = 0$ OR $y = 0$

A AND C $\quad y = 0$

$(x^2 - a)^2 = 0$

$x^2 - a = 0$

$x^2 = a$

$x = \pm\sqrt{a}$

$A = (-\sqrt{a}, 0)$

$C = (\sqrt{a}, 0)$

B $\quad x = 0$

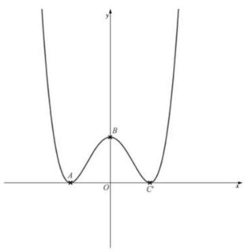
$f(x) = (0^2 - a)^2 = (-a)^2 = a^2$

$\boxed{A = (-\sqrt{a}, 0) \quad B = (0, a^2) \quad C = (\sqrt{a}, 0)}$

Q5b

5b

A sketch of the graph with equation $y = f(x)$, where $f(x) = (x^2 - a)^2$, with $a > 1$ is shown below.



The points A, B and C are points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A, B and C.

[2]

(b) Sketch the graph of $y = -\frac{1}{2}f(x-1)$, labelling the images of the three points A, B and C and stating their coordinates in terms of a.

[3]

(c) Suggest, in terms of $f(x)$, a combination of at least two transformations, such that the points A, B and C transform to new positions but remain lying on their respective axes.

[2]

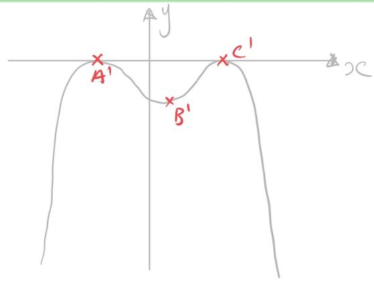
b) TRANSLATION $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

VERTICAL STRETCH SF $\frac{1}{2}$

VERTICAL REFLECTION x AXIS

$A = (-\sqrt{a}, 0) \quad B = (0, a^2) \quad C = (\sqrt{a}, 0)$

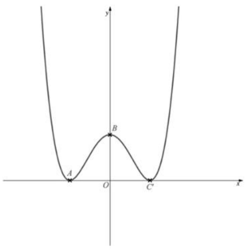
$\boxed{A' = (1-\sqrt{a}, 0) \quad B' = (1, -\frac{1}{2}a^2) \quad C' = (1+\sqrt{a}, 0)}$



Q5c

5c

A sketch of the graph with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x^2 - a)^2$, with $a > 1$ is shown below.



The points A, B and C are points where the graph intercepts the coordinate axes.

(a) Write down, in terms of a, the coordinates of A, B and C.

[2]

(b) Sketch the graph of $y = -\frac{1}{2}\mathrm{f}(x-1)$, labelling the images of the three points A, B and C and stating their coordinates in terms of a.

[3]

(c) Suggest, in terms of $\mathrm{f}(x)$, a combination of at least two transformations, such that the points A, B and C transform to new positions but remain lying on their respective axes.

[2]

c) ANY COMBINATION OF STRETCHES INCLUDING REFLECTIONS WHERE $a, b < 0$

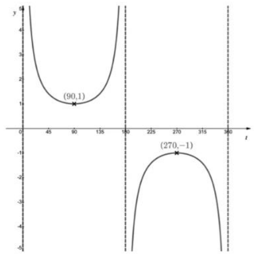
$$y = af(bx) \quad a, b \neq 0$$

$$\boxed{y = 3f(-5x)}$$

Q6a

6a

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \operatorname{cosec} t$, $0° \leq x \leq 360°$.



The vertical distance between the minimum point, $(90, 1)$, and the maximum point, $(270, -1)$ is 2. The horizontal distance between them is 180.

(a) Find, in terms of a, the vertical and horizontal distances between the minimum and maximum point on the graph of $y = -\frac{1}{a}\mathrm{f}(at)$, $a \neq 0$.

[4]

(b) Hence or otherwise show that the distance between the minimum and maximum point on the graph of $y = -\frac{1}{a}\mathrm{f}(at)$, $a \neq 0$, is

$$\frac{2\sqrt{8101}}{a}$$

[2]

a) $-\frac{1}{a}\left(\operatorname{Cosec} \frac{1}{a}t\right)$

DISTANCE

$-\frac{1}{a} = \left|\frac{1}{a}\right|$

HORIZONTAL DISTANCE

$$180 \times \frac{1}{a} = \frac{180}{a}$$

$$\boxed{\left|\frac{180}{a}\right|}$$

VERTICAL DISTANCE

$$2 \times -\frac{1}{a} = -\frac{2}{a}$$

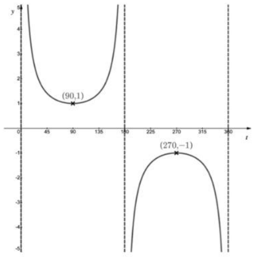
$$\boxed{\left|\frac{2}{a}\right|}$$

MODULUS ENSURES DISTANCE IS POSITIVE

Q6b

6b

The diagram shows the graph of $y = \mathrm{f}(t)$, where $\mathrm{f}(t) = \operatorname{cosec} t,\ 0° \le x \le 360°$.



The vertical distance between the minimum point, $(90\,,1)$, and the maximum point, $(270\,,-1)$ is 2. The horizontal distance between them is 180.

(a) Find, in terms of a, the vertical and horizontal distances between the minimum and maximum point on the graph of $y = -\frac{1}{a}\mathrm{f}(at), a \ne 0$.

[4]

(b) Hence or otherwise show that the distance between the minimum and maximum point on the graph of $y = -\frac{1}{a}\mathrm{f}(at), a \ne 0$, is

$$\frac{2\sqrt{8101}}{a}$$

[2]

b) HIDDEN PYTHAGORAS

$$\sqrt{\left|\frac{2}{a}\right|^2 + \left|\frac{180}{a}\right|^2}$$

$$\sqrt{\frac{4}{a^2} + \frac{32400}{a^2}}$$

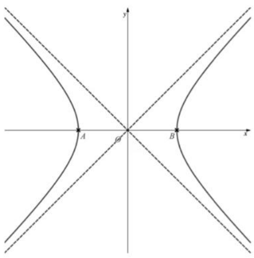
$$\sqrt{\frac{32404}{a^2}} = \frac{\sqrt{32404}}{a}$$

$$\boxed{\frac{2\sqrt{8101}}{a}}$$

Q7a

7a

The diagram below shows the graph of $x^2 - y^2 = 1$.



The graph has asymptotes with equations $y = x$ and $y = -x$.
The graph intercepts the x-axis at the points A and B.

(a) (i) Write down the coordinates of points A and B.
(ii) Determine the coordinates of the points A and B on the graph of $(x-3)^2 - (y-1)^2 = 1$.

[4]

(b) (i) Determine the equations of the asymptotes on the graph of $(x-3)^2 - (y-1)^2 = 1$.
(ii) Find the domain of $(x-3)^2 - (y-1)^2 = 1$.

[5]

a) i) $y = 0$

$x^2 - 0 = 1$

$x^2 = 1$

$x = \pm 1$

$\boxed{A = (-1, 0) \quad B = (1, 0)}$

ii) NOT IN FORM $f(x)$

TRANSLATION $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $(x-3)$

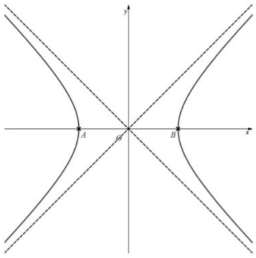
TRANSLATION $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $(y-1)$

$\boxed{A'(2,1) \qquad B'(4,1)}$

Q7b

7b

The diagram below shows the graph of $x^2 - y^2 = 1$.



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(a) (i) Write down the coordinates of points A and B.
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(b) (i) Determine the equations of the asymptotes on the graph of $(x-3)^2 - (y-1)^2 = 1$.
(ii) Find the domain of $(x-3)^2 - (y-1)^2 = 1$. [5]

b) i) $y = x$

$y - 1 = x - 3$ $\boxed{y = x - 2}$

$y = -x$

$y - 1 = -(x-3)$ $\boxed{y = 4 - x}$

ii) USING x COORDINATES OF A' B'

$x \in \mathbb{R}$ $x \leq 2$ $x \geq 4$

OR

$x \in \mathbb{R}$ $|3 - x| \geq 1$